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A SIMPLE GEOMETRICAL EXPLANATION OF CERTAIN APPARENTLY ABNORMAL FORMS OF THE SHADOW OF A SATELLITE DURING ITS TRANSIT ACROSS THE DISK OF *JUPITER*.

By J. M. Schaeberle.

The curious change of form which the shadow of a satellite undergoes during its transit across the disk of *Jupiter* has often been noticed and commented upon, but, so far as I am aware, no satisfactory explanation has been given.

The solution of the problem which I now give is of such amazing simplicity that all doubt as to its validity is removed, the proof involving only simple geometrical principles.

Let R denote the radius of *Jupiter;*
" r " " satellite radius of the shadow cone where it intersects the surface of *Jupiter.*

Then for a central transit this shadow cone intersects the surface of *Jupiter* in an oval, the linear length d_o of the transverse diameter being given by the expression

$$d_o = R \cdot 2 \sin^{-1} \left(\frac{r}{R}\right) \tag{1}$$

while the longitudinal diameter d varies from $d = d_o$ near the center of the disk to a maximum value $d = d_m$ given by the expression

$$d_m = R \cdot \text{covers}^{-1} \left(\frac{2r}{R}\right) \tag{2}$$

For central transits no errors will be introduced if, for greater simplicity, we use the chords d'_o and d'_m in place of the arcs.

The actual linear lengths of these chords are

$$d'_o = 2 \text{ R} \sin \left(\frac{r}{R}\right) \tag{3}$$

$$d'_m = 2 \text{ R} \sin \tfrac{1}{2} \left(\text{covers}^{-1} \left(\frac{2r}{R}\right)\right) \tag{4}$$

As seen from the Sun, these chords are of the same length in projection, since the one corresponding to d_o is inclined 90° to the planet's radius-vector, while the other makes the angle $\frac{1}{2}$ covers^{-1} $\left(\frac{2r}{R}\right)$ with the same line ; or

$$2\ R \sin\left(\frac{r}{R}\right) = 2\ R \sin^2 \tfrac{1}{2}\left(\text{covers}^{-1}\left(\frac{2r}{R}\right)\right) \qquad (5)$$

Now let θ denote the angular distance between the Earth and Sun as viewed from *Jupiter;* then the two limiting angles which the shadow chord makes with a line drawn to the Earth are

$$\theta + \tfrac{1}{2}\ \text{covers}^{-1}\left(\frac{2r}{R}\right) \text{ and } \theta - \tfrac{1}{2}\ \text{covers}^{-1}\left(\frac{2r}{R}\right)$$

The plus sign always refers to the chord at the visible terminator and the minus sign to the chord at the invisible terminator of *Jupiter.* Multiplying the actual length of each chord by the sine of its angle of inclination to the line of sight, we obtain the observed maximum (and minimum) values of the longitudinal diameters r'_m of the shadow

$$r'_m = 2\ R \sin \tfrac{1}{2}\left(\text{covers}^{-1}\left(\frac{2r}{R}\right)\right) \sin\left(\theta \pm \tfrac{1}{2}\ \text{covers}^{-1}\left(\frac{2r}{R}\right)\right) \quad (6)$$

When $\theta = 0$, this expression becomes $r'_m = r_0$.

The minimum value of r'_m can never be observed, as the shadow can only be seen when its angular distance from the invisible terminator exceeds the angle θ.

The interpretation of equation (6) leads to the following fundamental law, which is almost self-evident.

Near the visible terminator of a superior planet the observed longitudinal diameter of the shadow of a satellite is always greater than the transverse diameter, and just the reverse is true when the shadow is near the invisible terminator; the two principal axes of the oval approach equality as the distance from the center of the disk diminishes. The distortions reach a maximum when the planet is in quadrature with the Sun. There are no distortions when $\theta = 0°$ or $\theta = 180°$.

Let us now apply these formulæ to the case of *Jupiter's* first satellite. From observation we know that very slight contrasts of light and shade in the case of *Jupiter's* satellite produce very marked contrasts when these are projected against the disk of *Jupiter;* consequently the umbra and more or less of the penumbra are involved in the diameter of the shadow observed, the effective diameter being a function of the brightness of the background against which the shadow is projected. Taking the effective diameter of the shadow at 2100 miles, we find from equation (1) that the actual length of the shadow near the terminator is about 14,500 miles, and the chord is inclined to the planet's radius-vector 9°.5. When *Jupiter* is in quadrature, θ is something over 11°; therefore, the chord 14,500 miles long is seen under the maximum angle 20.5. Consequently, its length projected on a plane normal to the line of sight is more than 5000 miles, or more than twice the vertical diameter of the shadow. When the shadow(complete) is internally tangent to the *full* outline of the planet, the observed horizontal diameter of the planet is, for the same value of θ, but little more than one-fourth of the observed vertical diameter. It is probable that the assumed effective diameter, 2100 miles, is much too large for the shadow. If a less value is used, the distortion becomes increasingly greater. If the transit is not central, the distortions will be oblique near the terminators, and on the central meridian the axes will be slightly unequal. The exact geometrical form can now, of course, be predicted.*

PHOTOGRAPHIC VERIFICATION OF THE MOST DELICATE OPTICAL DETAILS ON THE MOON.

By Professor L. Weinek.†

Professor L. Weinek, Director of the Imperial Observatory of Prague, has submitted to the Imperial Academy of Sciences of Vienna, as a continuation of his recent lunar studies, drawings of the ring-plane *Capella* and of the crater *Taruntius* C, enlarged forty times from photographs taken at the Lick Observatory. His explanatory remarks are as follows:

Drawings I and II represent the interior of the ring-plane *Capella;* drawings III, IV and V show the small crater on the north wall of *Taruntius*, which Maedler calls C.

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* When the elongated shadow falls centrally upon a very narrow bright belt or spot, an apparent contraction of the penumbral portion of the shadow will have a tendency to form an apparent double shadow, especially when the seeing is not first-class. This explanation requires that the two portions of the shadow shall always lie nearly along a radius, or along a line parallel to the circumference. In the former position the shadow will be near the visible terminator and appear larger than in the latter position, which will be near the invisible terminator of the planet. J. M. S.

† Translated for the Society by F. R. Ziel, Secretary and Treasurer of the A. S. P., from the *Sitzungsbericht* of the Imperial Academy of Sciences at Vienna, July 6, 1893.